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Subdiffractive light pulses in photonic crystals

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We investigate propagation of light pulses in photonic crystals in the vicinity of the zero diffraction point. We show that Gaussian pulses due to nonzero width of their temporal spectrum spread weakly in space and time during the propagation. We also find the family of nonspreading pulses, propagating invariantly in the vicinity of the zero diffraction point of photonic crystals.

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I. INTRODUCTION

Since the initial proposal of the concept of photonic crystals (PCs) in 1987 [1], a number of studies revealed that these materials with a periodic modulation of the refraction index on a spatial scale of the order of the wavelength of light, are powerful tools to control and modify the propagation of electromagnetic fields. PCs offer the possibility of engineering the dispersion properties of light to yield photonic band gaps in the transmission and reflection spectra; thus, they can be designed to act as light conductors or insulators [2]. PCs also modify (in particular, reduce strongly) the phase and group velocities of light [3]. Recently it became apparent that PCs can also modify the diffraction of light, in that the diffraction can become negative if the refractive index is modulated in a direction perpendicular to the propagation of the light (one-dimensional PCs) [4]. Negative diffraction was also predicted for acoustic [5] and matter waves [6,7] in one-dimensional periodic materials.

If the diffraction is positive at one edge of the propagation band and is negative at the other edge, an inflection point inside of the phonic band can be expected, characterized by the vanishing diffraction. The vanishing of the diffraction has been shown for the arrays of waveguides [8] and in the resonators with periodic modulation of refractive index in one transverse direction [9]. The vanishing of diffraction means that at some particular point of parameter space and for a given frequency, the curvature of the spatial dispersion curve $1/2(\partial^2 k_\parallel/\partial k_\perp^2)$ becomes zero (here $k_\parallel$ is the longitudinal and $k_\perp$ is the transverse component of the wave vector). The nondiffractive propagation of the monochromatic light beams in two-dimensional photonic crystals has been shown analytically [10], numerically [11], and experimentally [12] until now.

All the above studies consider diffraction management of monochromatic light beams. For pulses of nonzero width of the spectra, the diffraction, in the leading order, can disappear for a particular frequency only. The other frequency components, not corresponding exactly to the nondiffractive point, broaden diffractively in propagation. This can result in a complicated shaping of the pulse, propagating in the vicinity of the zero diffraction point. Also, the periodic modulation of refraction index introduces an additional group velocity dispersion [3], causing the temporal broadening of the pulses. Having in mind these two ingredients caused by PCs (strong dependence of diffraction on frequency and appearance of the group velocity dispersion), the pulse propagation can become very complicated.

Theoretically the pulse propagation in two-dimensional PCs has been studied in [13], where the general formalism of the nonmonochromatic beam propagation has been developed. The propagation invariant wave packages were also suggested in [14], in the regimes of anomalous diffraction, i.e., at minima and maxima of the frequency surface (corresponding to the band edges) and at the saddle points. The present article is devoted to the study of the pulse propagation in the vicinity of the zero diffraction point, i.e., at the inflection points of the frequency surface. First we analyze the pulse propagation close to the zero diffraction point for PCs and develop the normal form description close to this point (Sec. III). Next, basing on the normal form description, we derive the amplitude equations for the evolution of the macroscopic spatiotemporal shapes of the pulses (Sec. IV). The spatiotemporal shapes in this macroscopic description are defined on the large spatiotemporal scales: the small spatiotemporal scales associated with the scale of refraction index modulation in PCs are eliminated. In Sec. V we calculate numerically the evolution of the spatiotemporal shape of the pulse and evaluate the (weak) spatiotemporal broadening close to the zero diffraction point, basing on the amplitude equations. In addition, we perform the calculations under full, microscopic model in order to check the validity of the amplitude equation approach. Finally, in Sec. VI we propose the families of the pulses with particular spatiotemporal form, which do not broaden in space and time during the propagation. Such nondiffractive pulses in PCs are similar to the X pulses in homogeneous dispersive and diffractive materials [15], and to the recently studied X waves in periodic materials [7,14]. The X pulses have particular envelopes of spatiotemporal spectra, such that the dephasing of the different spatial components due to diffraction and dispersion mutually compensates and the pulses propagate without broadening. Analogously, the nonspreading pulses, as suggested by us, have specific envelopes of the spatiotemporal spectra in the vicinity of the zero diffraction point, which mutually

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compensate dispersion and (weak) diffraction in propagation through PCs.

II. MODEL

We consider two-dimensional PCs, consisting of superposition of two periodic, harmonic (i.e., sinusoidal) lamellae-like refraction index gratings: $\Delta n(\mathbf{r})=2m[\cos(\mathbf{q}_1\mathbf{r})+\cos(\mathbf{q}_2\mathbf{r})]$ with $|\mathbf{q}_1|=|\mathbf{q}_2|=q$ at angles $\pm\alpha$ to the optical axis. This resulting refraction index profile is $\Delta n(x,z)=4m\cos(q_\perp x)\cos(q_\parallel z)$, with $q_\parallel=q\cos(\alpha)$ and $q_\perp=q\sin(\alpha)$. The crystallographic axes of such PCs are $\pi\cdot(\pm 1/q_\perp, 1/q_\parallel)$, and the reciprocal lattice vectors are $\mathbf{q}_1$ and $\mathbf{q}_2$. We describe the light propagation under approximation of slowly varying (in space and time) envelopes [16]:

$$\left[\frac{1}{c}\frac{\partial}{\partial t}+\frac{\partial}{\partial z}-\frac{i}{2k_0}\frac{\partial^2}{\partial x^2}-i\Delta n(x,z)k_0\right]A(x,z,t)=0. \qquad (1)$$

Here $A(x,z,t)$ is the complex envelope of the electromagnetic field $[E(x,z,t)=A(x,z,t)e^{ik_0z-i\omega_0t}]$, with a wave number $k_0=\omega_0/c$ defined in two-dimensional space (x,z) and propagating along the z direction.

We consider next the reference frame moving with the velocity of light; thus, the first term of Eq. (1) disappears, and the refraction index becomes a function periodic in space and time, $\Delta n(x,z)\rightarrow\Delta n(x,z-ct)$.

III. MONOCHROMATIC CASE: BLOCH MODE EXPANSION

First we perform an analytical study of the propagation of the plane monochromatic waves [time independent limit of (1)] by expanding the electromagnetic field into a set of spatially harmonic modes, in a similar way as, e.g., described in [17,10],

$$A(x,z)=\sum_{j,l}A_{j,l}e^{ik_{\perp,j}x+ik_{\parallel,l}z}, \qquad (2)$$

where $\mathbf{k}_{j,l}=(k_{\perp,j},k_{\parallel,l})=(k_\perp+jq_\perp,k_\parallel+lq_\parallel)$, $j,l=\ldots,-1,0,1,\ldots$. The expansion results in a coupled system for the amplitudes of harmonics,

$$(-2k_0k_{\parallel,l}-k_{\perp,j}^2)A_{j,l}+2mk_0^2\sum_{u=j\pm1,v=j\pm1}A_{u,v}=0 \qquad (3)$$

Solvability of (3) results in transverse dispersion relation (the dependence of the longitudinal component $k_\parallel$ on the transverse component $k_\perp$ of the wave vector of the Bloch mode), which, as calculated numerically, is given in Fig. 1(a). In the limit of the vanishing refractive index modulation $m\rightarrow0$, the formal solution of (3) consists of a set of parabolas (dashed curves in Fig. 1) shifted one with respect to another by the reciprocal vectors of the PC lattice $\mathbf{q}_{1,2}$. They represent the transverse dispersion curves for the uncoupled harmonic components of the expansion (2). In full (nonparaxial) description of the light propagation, these parabolas are to be substituted by corresponding circles. The modulation of the refractive index $m\neq0$ lifts the degeneracy at the crossing points and gives rise to the band gaps in spatial wave number domain [Fig. 1(a)]. For particular amplitude of the modulation given by m, for a particular geometry of PC given by $\mathbf{q}_{1,2}$ and for a particular frequency $\omega_0=k_0c$, the plateaus on the transverse dispersion curves appear, indicating the vanishing of diffraction. Figure 1(b) indicates the vicinity of the zero diffraction point curve at $k_\perp=0$.

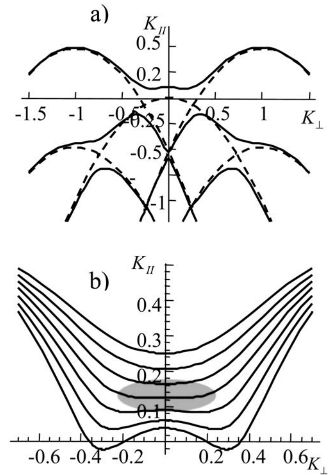

FIG. 1. The transverse dispersion curve as obtained by numerical solution of (3) using the expansion in five harmonics: (a) all five dispersion curves (corresponding to five families of eigenmodes), for $f=0$ (dashed) and $f=0.15$ (solid); (b) upper dispersion curve for different values of the frequency $\delta\omega=-0.2,-0.1,0,0.1,0.2$ from the bottom to the upper curve for $f=0.2$. PC geometry parameter $Q_\parallel=0.479$.

We perform an asymptotic analytical description of the diffraction curves at the zero diffraction point by considering only the three most relevant expansion modes in (2), those with $(j,l)=(0,0),(-1,-1),(1,1)$. The three mode description is valid in the limit of small modulation, as shown in [10]. Also, Fig. 1 shows that the deformation of spatial dispersion curve and the appearance of straight segments occurs essentially due to intersection (and the lift of degeneracy) between the three modes (three dispersion curves). The presence of the other ones becomes sensible in case of strong interaction (large values of m), which is not the case studied in the present paper. We introduce a set of adimensional parameters by the following normalization for the wave numbers of light: $K_\perp=k_\perp/q_\perp$, $K_\parallel=2k_\parallel k_0/q_\perp^2$. The space coordinates are then rescaled as $Z=zq_\perp^2/2k_0$ and $X=xq_\perp$. Two significant parameters remain after the normalization: $f=2mk_0^2/q_\perp^2$, which represents the modulation depth of the Bloch mode in the PC, and $Q_\parallel=2q_\parallel k_0/q_\perp^2$, which is proportional to the angle between the crystallographic axes of the PC (geometry parameter). The asymptotic solution of (3) in terms of normalized parameters is (see also [10])

$$K_\parallel=\frac{2f^2}{(1-Q_\parallel)}+K_\perp^2\left(\frac{8f^2}{(1-Q_\parallel)^3}-1\right). \qquad (4)$$

The following smallness conditions are to be fulfilled for the parameters: $1-Q_\parallel=O(\varepsilon)$ and $f=O(\varepsilon^{3/2})$ for the validity of (4). The smallness condition for $K_\perp$ can be chosen at this stage as $K_\perp = O(\varepsilon^\alpha)$ (with $\alpha > 1$) in order to consider (4) as the truncation of asymptotic expansion. However, seeking for a consistence with the subsequent expansions we fix the smallness condition $K_\perp = O(\varepsilon^2)$, which results in the higher order terms in (4) of $O(\varepsilon^6)$. The first neglected term in (4) reads $K_\perp^4 32 f^2/(1-Q_\parallel)^5$ (see [10]), i.e., it is associated with the second order diffraction.

The expression for the zero diffraction curve follows directly from (4) as eliminating the dependence of $K_\parallel$ on transverse wave vector $K_\perp$,

$$8f^2 = (1 - Q_\parallel)^3. \tag{5}$$

In general, the zero diffraction curve exists as the single valued function $Q_\parallel(f)$ for all values of $0 < f < \infty$ in the plane $(f, Q_\parallel)$, as can be obtained by numerical solution of (3) by using a sufficiently large number of expansion modes [10].

IV. NONMONOCHROMATIC CASE

The zero diffraction relation (5) in terms of initial variables reads $8(2mk_0^2/q_\perp^2)^2 = (1 - 2q_\parallel k_0/q_\perp^2)^3$, i.e., relates the parameters of the PC $(m, q_\parallel, q_\perp)$, and the frequency of monochromatic wave $\omega_0 = k_0 c$. Obviously, the zero diffraction condition for a fixed parameter PC holds for a given frequency only. We consider next a small variation of the carrier frequency $\omega = \omega_0(1 + \delta\omega)$, resulting in the variation of the normalized variables in (4): $Q_\parallel = Q_{\parallel,0}(1 + \delta\omega)$, and $f \to f_0(1 + \delta\omega)^2$ (the values $Q_{\parallel,0}$ and f_0 here correspond to the zero diffraction). Retaining smallness conditions for $1 - Q_\parallel$, f, and $K_\perp$ as in (4), imposing the following smallness condition for the width of spectral width of the pulse $\delta\omega = O(\varepsilon^2)$ and collecting the terms in (4) up to order $O(\varepsilon^5)$, we obtain

$$K_\parallel = K_{\parallel,0} + \frac{\delta\omega}{V_0} + \frac{\delta\omega^2}{4} + \alpha[\delta\omega K_\perp^2 + O(\varepsilon^6)]. \tag{6}$$

Here $K_{\parallel,0} = (1 - Q_{\parallel,0})^2/4$, $1/V_0 = (1 - Q_{\parallel,0})(3 - Q_{\parallel,0})/2$, and $\alpha = 3/(1 - Q_{\parallel,0})$. The smallness parameter ε is a measure of the pulse duration (normalized to the period of longitudinal modulation of the refraction index) and of the width of the beam (normalized to the period of transverse modulation of the refraction index), and is specified by particular scalings, as given above. The different (right-hand side) rhs terms of (6) mimic the different ingredients of light propagation in the PC: a constant shift of the longitudinal wave number (term 1) of order of $O(\varepsilon^2)$, a change (decrease) of the group velocity (term 2) of order of $O(\varepsilon^3)$, the PC induced group velocity dispersion of the "normal" sign (term 3) of order of $O(\varepsilon^4)$, and the diffraction dependence on the frequency in the vicinity of the zero diffraction point (term 4) of order of $O(\varepsilon^5)$, which is an exceptional ingredient of the materials with vanishing diffraction (PCs in this case) close to the inflection point. The next term, which we neglect in (6), is of the order of $O(\varepsilon^6)$ and is associated with the second order diffraction, i.e., it is proportional to $K_\perp^4$.

The different scaling for $\delta\omega$ and $K_\perp$ corresponds to different durations of pulse and the width of beam and generates a different dispersion relation (6). We chose the particular scalings in order to derive Eq. (6) in a systematic way, with the terms following from normal form analysis at the vicinity of the inflection point. We note that (6) [and subsequently (7)] can be derived phenomenologically by inspecting the zero diffraction (inflection) point. One then can expand formally the longitudinal wave vector $K_\parallel$ in power series of $K_\perp$ and $\delta\omega$, using particular smallness assumptions, and keeping the terms of the leading order. This results in an equation with the same structure as (6), however, with undefined coefficients of the rhs terms. One more possibility to arrive at (6) is to adapt the general formalism developed in [13] to our specific case, i.e., to perform the expansions around the zero diffraction point of the PCs.

Next, rewriting Eq. (6) in terms of time and space variables ($\partial/\partial Z \leftrightarrow iK_\parallel$, $\partial/\partial X \leftrightarrow iK_\perp$, $\partial/\partial T \leftrightarrow -i\delta\omega$, where the adimensional space variables are as normalized above and the adimensional time is $T = \omega_0 t$), we obtain

$$\left(\frac{\partial}{\partial Z} - iK_{\parallel,0} + \frac{1}{V_0}\frac{\partial}{\partial T} + \frac{i}{4}\frac{\partial^2}{\partial T^2} - \alpha\frac{\partial}{\partial T}\frac{\partial^2}{\partial X^2}\right)A = 0 \tag{7}$$

which is the central equation for our analysis. Under the approximations used to derive (6) and (7), the function $A(X, Z, T)$ can be considered as the slow (in space and time) envelope of the Bloch function at zero diffraction point. Strictly speaking, $A(X, Z, T)$ is the envelope of Wannier functions, as could be proved by applying the analysis in [14]; however, in approximations used by us, the envelopes of corresponding Wannier functions and of Bloch function coincide.

V. GAUSSIAN PULSES

Next we analyze the propagation of the Gaussian pulses solving numerically Eq. (7). In fact, due to the linear character of (7) we calculated the shape of the pulse in spatiotemporal Fourier domain, solved analytically (6), and recovered numerically the shape in space-time domain of the propagated pulse. The typical evolution of initially Gaussian pulse is given in Fig. 2. In general, we obtain an expected result that the initially Gaussian beam pulse distorts during the propagation due to the dependence of diffraction coefficient on the frequency [term 4 in the rhs of (6) and (7)].

The overall temporal (spatial) broadening of the pulsed beam can be estimated by analyzing the propagation of the separate frequency components of the radiation independently and by averaging over spatial (temporal) frequency. The result is that the spatiotemporal broadening is characterized by two PC induced dispersion/diffraction lengths: (i) dispersion length: $L_{disp,1} = 4T_0^2$ depending on pulse duration only and responsible for the symmetric broadening of the pulse in time and (ii) the mixed dispersion and/or diffraction length $L_{disp,2} = X_0^2 T_0/\alpha$, resulting in a spatiotemporal broadening and distortion of the pulse. We note that the diffraction (Rayleigh) length of the beam in free propagation (absence of refraction index modulation) is $L_{Rayleigh} = X_0^2$ under normalizations used. This estimate leads

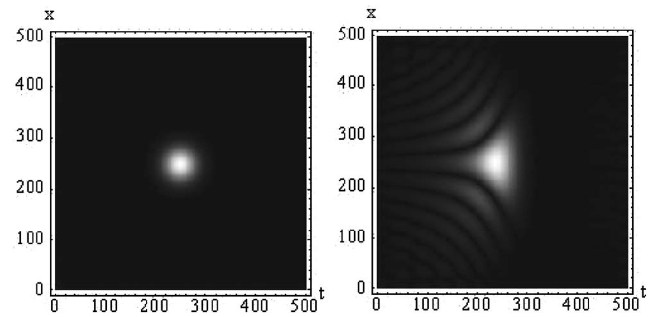

FIG. 2. Propagation of the Gaussian pulse centered around the zero diffraction point, as obtained by numerical integration of (7) with $Q_{\parallel}=0.44$ ($\alpha=5.4$), $f=0.15$. The initial Gaussian shape is on the left and the pulse propagated over $Z=120$ is on the right. The half-width of the initial Gaussian pulse in Fourier spatial and temporal domain is $\Delta K_{\perp}=\Delta\delta\omega=0.2$, resulting in the half-width in the normalized space-time domain of $X_0=T_0=6.2$.

to the conclusion that the short pulses of duration $T_0<\alpha$ diffract stronger (at this zero diffraction point) than the corresponding monochromatic Gaussian beam propagating in the homogeneous space.

Figure 3 shows the evolution of the width and the duration of the pulse as obtained by numerical integration of (7). The calculated spatial and temporal broadening of the pulse is in accordance with the above estimates. For example, the mixed dispersion and/or diffraction length is $L_{disp,2}\approx 40$ and essentially determines the spatial and temporal broadening of the pulse as being significantly shorter than the PC induced dispersion length $L_{disp,1}=160$ under the conditions used.

In order to justify the validity of the above results of the Gaussian pulse evolution based on the integration of the amplitude equation (7), we also performed the corresponding calculations by numerically integrating the initial microscopic model (1). We used the split-step integration technique by calculating the diffraction operator in the spatiotemporal Fourier domain and calculating the phase evolution due to refraction index modulation in space-time domain on the grid of (256×256) points. The integration results are given in Fig. 4, where the misshaping of the spatiotemporal pulse

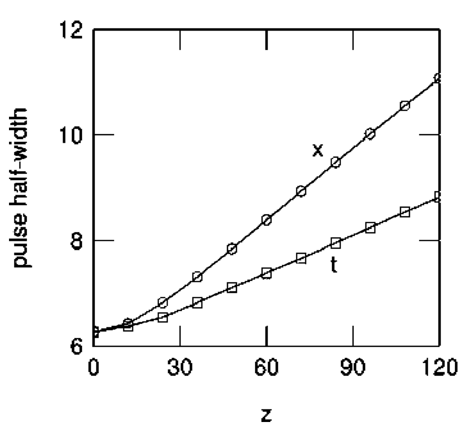

FIG. 3. The evolution of the width and the duration of initially Gaussian pulse. Parameters and conditions as in Fig. 2.

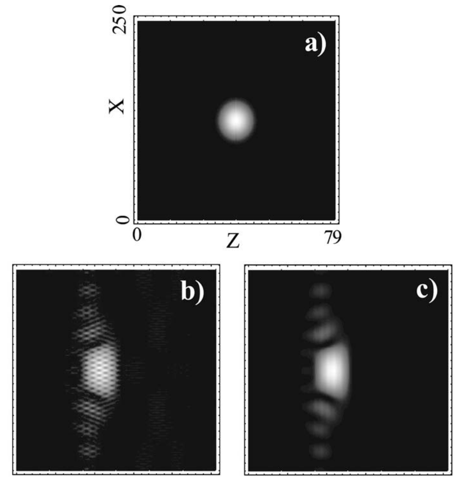

FIG. 4. Propagation of the Gaussian pulse centered around the zero diffraction point, as obtained by numerical integration of (1). The same conditions as in Fig. 2, with $Q_{\parallel}=0.57$, $f=0.15$: (a) initial Gaussian shape (the half-widths of the pulse are $X_0=6$ and $T_0=4.5$), (b) spatiotemporal envelope of the pulse propagated over the distance $Z=240$; and (c) Fourier filtered envelope of the final pulse (with the high spatiotemporal Fourier components removed).

profile is evident and is in qualitative accordance with Fig. 2. Both figures agree in demonstrating the following features: (i) that (small) diffraction occurs for short pulses, in spite of nondiffractive behavior, for monochromatic waves under the same parameters; (ii) that this (the unusual) diffraction gives rise to fringes, whereas the normal diffraction would just result in a broadening; and (iii) that these fringes are placed asymmetrically along the pulse in time, which means that their appearance (i.e., the diffraction) is related with the temporal operators (the derivation with respect to time). The broadening of the envelope also quantitatively agrees with the above calculated mixed dispersion and/or diffraction lengths. We note that the calculations in both cases have been compared with $f=0.15$, which is beyond the smallness conditions as used to obtain the envelope expansions (6) and (7). The longitudinal coordinate, instead of retarded time, was used in Fig. 4, which relates to retarded time as $T=2Zk_0^2/q_{\perp}^2$.

One set of the real-world parameters to a particular set of normalized parameters used in Fig. 2 and in Fig. 4 is the following: $\lambda=0.5$ μm, $q_{\perp}=0.5k_0$, $q_{\parallel}=0.08k_0$ ($\lambda_{\perp}=1$ μm and $\lambda_{\parallel}=6.3$ μm, respectively), $m=0.0185$. Then the parameters of the pulses are $x_0\approx 6$ μm, $\tau_0\approx 17$ fs, and the propagation distance is $z\approx 90$ μm for Fig. 2 and $z\approx 180$ μm for Fig. 4.

VI. NONDIFFRACTIVE PULSES

We searched for particular spatiotemporal shapes of the pulses which are invariant under the propagation described by (7). We note that in the frames of the full model (1) the invariant pulses are in fact oscillatory on the small space-time scale (i.e., they move through periodic potential); therefore, the direct search of the invariant structures is compli-

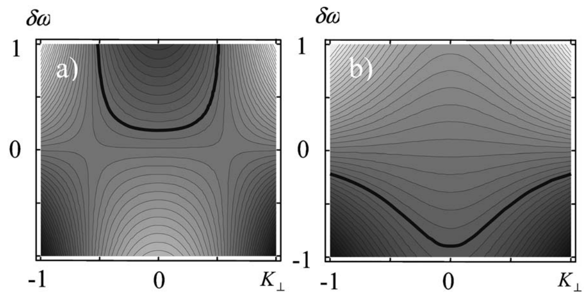

FIG. 5. Isolines $K_{\parallel}$=const as calculated from (8) for given values of the velocity: $\Delta V=(1/V_0-1/V_1)=-10$ (a), and $\Delta V=+10$ (b), for $Q_{\parallel,0}=0.9$. The bold isolines are used for preparation of propagation invariant pulses in subsequent figures.

cated. The pulse propagating invariantly with the group velocity V_1 is obtained directly from (6),

$$K_{\parallel,1}+\frac{\delta\omega}{V_1}=K_{\parallel,0}+\frac{\delta\omega}{V_0}+\frac{\delta\omega^2}{4}+\alpha\delta\omega K_{\perp}^2. \quad (8)$$

Here $K_{\parallel,1}$ is a free parameter (having a sense of longitudinal wave number) spanning the one-parameter family of the pulses propagating with a given group velocity V_1. Figure 5 shows the families of the isolines of $K_{\parallel,1}$ in the plane of $(\delta\omega,K_{\perp})$ obtained for two different group velocities. The radiation modes within a given isoline propagate with the same wave vector $K_{\parallel,1}$; therefore, the field formations belonging to each separate isoline propagate without dispersive/diffractive broadening.

We note that so called X pulses are propagation invariant structures residing on hyperbolas in the space-time Fourier domain [15]. We also obtain a big variety of invariant structures that are mostly of bell shape in space-time Fourier domain (Fig. 5). We generate several typical invariant structures. For this purpose we fix the desired frequency in (8), select a particular isoline $K_{\parallel}$=const (i.e., one isoline from Fig. 5), and generate a spatiotemporal spectrum of the pulse around a selected isoline. The selection of the infinitely narrow spectra $\delta K_{\parallel}\rightarrow 0$ results in tails of the pulse expanding to infinity, i.e., to so called algebraically localized pulses. Therefore, we generated the spatiotemporal spectrum of the finite $\delta K_{\parallel}$ centered around a particular isoline. We perform the inverse Fourier transformation and obtain the spatiotemporal shapes of such propagation invariant pulses as shown in Fig. 6(a) and 6(b). The finite spatiotemporal spectrum, on one hand, results in finite in space and time pulses (i.e., with exponential localization) and, on the other hand, results in weak spreading of the pulses (more precisely, in the spreading of the exponentially localized part of the pulses) during the propagation. The distance at which the exponentially truncated tails of the pulses spread sensibly (an analog of the Rayleigh length) can be evaluated by assuming that the spectral components dephase during that propagation, $\delta K_{\parallel}Z\approx 1$, and for pulses shown in Fig. 6 is $Z_{\mathrm{Re}l}\approx 70$. We note that this is related to the spreading of the cutoff radius of the tails; the central peak propagates without the broadening.

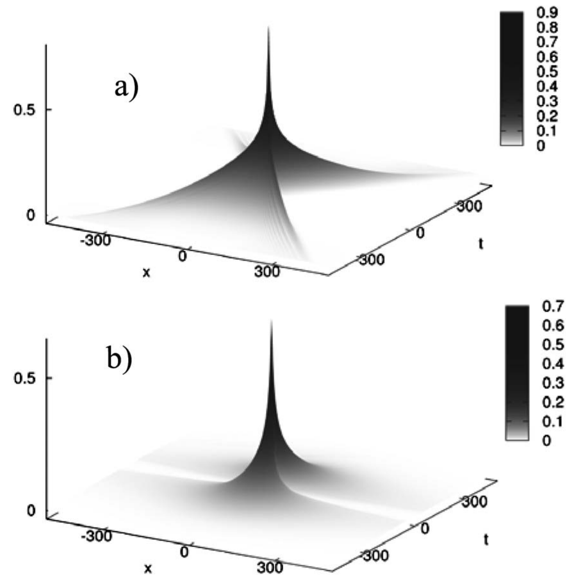

FIG. 6. Two invariant pulses as constructed by selecting radiation around the bold isolines from Fig. 5: (a) $V=10$, $K_{\parallel}=8.01$; and (b) $V=-10$, $K_{\parallel}=5.81$. The "thickness" of the isoline is $\delta K_{\parallel}=0.015$.

We calculate numerically the (invariant) propagation of the algebraically and exponentially localized pulses. We prepare the pulses as described above and integrate the evolution equation (7) numerically. The invariant propagation of algebraically localized pulses is trivial. The exponentially localized pulses propagate without a visible distortion for the propagation distances $Z\leqslant Z_{\mathrm{Re}l}$. For longer propagation the tails show distortion; however, the central peak remains narrow, as expected.

VII. SUMMARY AND CONCLUSIONS

We investigate the propagation of ultrashort light pulses in PCs in the vicinity of the zero diffraction point by deriving the amplitude equation for the evolution of the spatiotemporal envelope of the corresponding Bloch mode and by solving the derived equation. The first conclusion is that the spatiotemporal envelope of the initially Gaussian pulses might experience sensible distortion. This means that the use of ultrashort pulses in nondiffractive information transfer lines should be problematic: the short pulses not only spread in time due to PC induced dispersion, but also experience distortion of spatial profile of the beam on the propagation length of the order of $L_{disp,2}=X_0^2T_0/\alpha$. This also means that the application of ultrashort pulses to transfer the spatial light patterns through PC nondiffractive lines results in smearing of the patterns. The critical length of the pulse for nondiffractive propagation is $T_0\approx\alpha$. This conclusion has also been justified by the integration of the full microscopic model (1).

The second conclusion is that the above problem can be solved to some extent by using specially prepared X-like pulses. Such pulses with the algebraic localization propagate without any distortion along PCs, and the pulses with the exponential localization propagate with a weak distortion.

This distortion affects only the weak tails of the exponentially localized x-like pulses but not the central peak, which remains well localized over distances significantly larger than the Rayleigh lengths.

Nonlinear generalizations are also possible. The weak nonlinearity in PCs could play a twofold role: stabilizing the X pulses by ensuring their exponential localization, analogous to stabilization of conventional X waves [18], and spontaneous generation of the X pulse in photonic crystals, in analogy to the nonlinear X pulse generation in homogeneous nonlinear materials [19,20].

ACKNOWLEDGMENT

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[16] The above analysis, strictly speaking, is legitimate when the spatial period of refraction index modulation is by order of magnitude larger than the wavelength of the light radiation.

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